Analysis of Functions with Differentiation Calculus of a Single Variable X. Du

- Specific points on a graph:
  - **Critical values** - values of x for which $f'(x)$ is 0 or undefined
  - **Points of inflection** - points on a graph where $f''(x)$ changes from negative to positive or vice versa. Often occurs where $f''(x)=0$
- Determining local (relative) and absolute (global) **extrema**:
  - Check all critical values
  - Check the boundary of the interval (i.e. the bounds OR $\pm\infty$)
  - Check the behavior of the function between critical values (i.e. be careful - there isn't always a relative extreme at every critical point).
    - At any relative extrema, $f'(x)$ is 0 or undefined.
    - Converse of this statement is NOT true.
  - Relative extrema are *points*. Absolute extrema are *values* of $f(x)$.
  - Be careful - extrema might not always exist
- **First Derivative Test**:
  - Find the set of all critical values.
  - If $f'(x)$ changes from positive to negative at c, then (c, $f(c)$) is a local maximum.
  - If $f'(x)$ changes from negative to positive at c, then (c, $f(c)$) is a local minimum.
- **Second Derivative Test**: Find the set of critical points and plug them into $f''(x)$ to see whether or not the critical points represent maxima or minima. $f''(x)>0 \Rightarrow$ min. $f''(x)<0 \Rightarrow$ max. $f''(x)=0 \Rightarrow$ the test fails.
- Find intervals on which $f(x)$ is increasing or decreasing.
  - Find all critical values
  - $f(x)$ **increasing**: $f'(x)>0$ $f(x)$ **decreasing**: $f'(x)<0$
  - Test whether $f(x)$ is increasing or decreasing on the open intervals between critical points. Two ways:
    - Plug a point c in between two critical values and find $f'(c)$.
    - Let a and b be two critical values where $a < b$. Compare $f(a)$ and $f(b)$.
- Determining **concavity** of $f(x)$ on an interval.
  - Find all points of inflection. Be careful - not all values of c such that $f''(c)=0$ correspond to points of inflection.
  - Create open intervals between the points of inflection.
  - $f(x)$ is **concave up** when $f''(x)>0$ $f(x)$ is **concave down** when $f''(x)<0$
- Specific types of lines:
  - **Tangent** lines (touches function at one point). Slope = derivative at a point.
  - **Secant** lines (crosses function at two points). Find slope between two points.
  - **Normal** lines (crosses function perpendicularly). Slope = negative reciprocal of derivative
- Graphs: **PMZ** rule.
  - **P**oint of inflection on $f(x)$ = **M**ax/**M**in on $f'(x)$ = **Z**ero on $f''(x)$
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